

Distributed Formation Control for Strings of Nonlinear Autonomous Agents with Decoupled Dynamics

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Abstract—We introduce a novel distributed control architecture for a class of nonlinear dynamical agents moving in the string formation, while guaranteeing trajectory tracking and collision avoidance. An interesting attribute of the proposed scheme is the fact that its performance is scalable with respect to the number of vehicles in the string. The scalability is a consequence of the complete “decoupling” of a certain bounded approximation of the closed-loop equations, that allows the study of the closed-loop stability by performing solely individual, local analyses of the closed-loops stability at each agent, which in turn guarantee the aggregated stability of the formation.

I. INTRODUCTION

The problem of distributed control for strings of dynamical agents can be seen as a particular instance of flocking for multi-agent systems. The flocking problem has been intensively studied in the last decade for agents with linear [3], [4], [17], [14] or nonlinear [2], [18] dynamics. Generally speaking, two important features characterize the flocking behavior of autonomous agents: cohesion and collision avoidance. In multi-agent systems they are implemented as connectivity/topology preservation [20], [19], [16], [15] and collision avoidance [3], [4], [2], respectively. While these features are sufficient for reasonable flocking behavior, in the control of strings (also known as platooning) it is essential to add a supplementary requirement related to avoiding the amplification of oscillations through the formation, phenomenon known as *string instability* [7], [10], [8], [9]. Even if string instability is circumvented, supplemental problems may be caused by the so called accordion effect (or slinky effect) consisting of weakly attenuated oscillations and long settling times of the relative positions and velocities of the vehicles. Both string instability and the slinky effect are consequences of the notorious lack of scalability of networks of dynamical agents. This lack of scalability causes the performance of the control scheme to depend on the number of vehicles in formations and also on the vehicle’s position in formation. For the case of LTI agents, a novel distributed control architecture that can

guarantee string stability even for distance-based headways and can also guarantee perfect trajectory tracking (*i.e.* the complete elimination of the slinky effect) in the presence of bounded disturbances and communications delays, has been recently proposed in [5].

For many important applications in networks of dynamical agents, the regulated signals in trajectory tracking or synchronization problems represent relative measurements such as interspacing distances, relative velocities (with respect to neighboring agents) or phase differences between (neighboring) coupled oscillators. In this paper, we present preliminary results on a class of novel distributed control policies where the relative measurements (with respect to the neighboring agents) are used by the local sub-controllers in conjunction with the knowledge of the control actions of the sub-controllers at the neighboring agents. It turns out that the performance of the resulting distributed control schemes vastly outperforms the distributed architectures based solely on relative measurements.

In this work, the dynamical models of the agents are taken to be non-linear and time invariant, satisfying a global Lipschitz-like condition. Such models represent an effective framework for describing the dynamics of the transmission block of road vehicles, from the break/throttle controls to the vehicle’s position on the roadway. For illustrative simplicity, we only look at the scenario of identical agents for which the formation graph is a string, however the proposed scheme could be adapted to multi-tree (no self-loops) graphs, once that the relative errors are defined adequately in order to avoid the well-known formation rigidity problems [22]. It is noteworthy that the string formation alone has important applications, as it addresses the longstanding platooning problem, which is paramount to the autonomous vehicles industry.

The problem considered in this paper can be rephrased as a multi-agent flocking problem with collision avoidance. The literature on this topic is very rich and considers both directed or undirected, fixed or time-varying interconnection graphs. The objective of the control scheme is to achieve the synchronization of the trajectories of all agents in the formation with the trajectory of the leader agent. Such trajectory tracking must be achieved while ensuring zero (steady-state) errors of the regulated measures (in our case the relative speed between two consecutive agents) and while avoiding collisions, *i.e.* performing the needed longitudinal steering (brake/throttle) maneuvers that guarantee the avoidance of collision with the preceding vehicle.

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The approach taken here is based on the use of Artificial Potential Functions [3], [2] in the formulation of the *local* control laws. When compared to the state-of-the-art, our results represent a consistent extension of the ones in [3], [2], from at least two perspectives. Firstly, it guarantees stability, velocity matching (trajectory tracking) and collision avoidance even for *directed topologies*, as illustrated by the distributed control scheme reported here. Secondly, it achieves complete *scalability* with respect to the number of agents in formation [2] (which is a daunting requirement for platooning systems, even in the case of LTI agents) and also with respect to the connectivity of the communications graph.

By comparison, the main result in [2] imposes that all the minor matrices of the weighted Laplacian matrix associated with the interconnection graph are positive definite and lower bounded by a certain constant. This practically requires the maximization of the eigenvalues of the weighted Laplacian matrix, which can be interpreted as the maximization of the number of interconnections in the underlined graph (see [11]) together with the maximization of its diagonal elements (see Geršgorin disk theorem [12]). It is worth noting that the first requirement involves the transmission of the exact state of the leader to many agents in the formation, while the second requirement represents high local control gains.

The most interesting feature of our proposed scheme is the fact that it achieves complete *scalability* with respect to the number of vehicles in the string. The scalability does not result from the tuning of the local sub-controllers [6], as it is a structural property for the class of all stabilizing controllers and it is a consequence of the complete “decoupling” of a certain bounded approximation of the closed-loop equations. This allows the study of the closed-loop stability by performing solely individual, local analyses of the closed-loops stability at each agent, which in turn guarantee the aggregated stability of the formation.

The paper is organized as follows: in Section II we introduce the general framework and we formulate the platooning control problem. Section III provides a preliminary description of the novel distributed control architecture introduced in this work along with a first glimpse at the closed-loop dynamics “decoupling” featured by the control scheme. Section IV contains the main result as it delineates the guarantees for stability, velocity matching and collision avoidance.

II. GENERAL FRAMEWORK AND PROBLEM STATEMENT

A. Preliminaries

Definition 2.1: The σ -norm of a vector x is defined as

$$\|x\|_\sigma \stackrel{\text{def}}{=} \frac{1}{\sigma} \left[\sqrt{1 + \|x\|_2^2} - 1 \right] \quad (1)$$

with σ is a positive constant. This is a class \mathcal{K}_∞ function of $\|x\|_2^2$ and is differentiable everywhere.

Definition 2.2: A set Ω is said to be *forward invariant* with respect to an equation, if any solution $x(t)$ of the

equation satisfies:

$$x(0) \in \Omega \implies x(t) \in \Omega, \forall t > 0.$$

Definition 2.3: Artificial Potential Function (APF). The function $V_{k,k-1}(\cdot)$ is a differentiable, nonnegative, radially unbounded function of $\|z\|_\sigma$ satisfying the following properties:

- (i) $V_{k,k-1}(\|z\|_\sigma) \rightarrow \infty$ as $(\|z\|_\sigma) \rightarrow 0$,
- (ii) $V_{k,k-1}(\|z\|_\sigma)$ has a unique minimum, which is attained at $\|z\|_\sigma = \delta_k$, with δ_k being a positive constant.

B. The Problem: Trajectory Tracking of the String Formation

We consider a *homogeneous* group of n agents (e.g. autonomous road vehicles) moving along the same (positive) direction of a roadway, with the origin at the starting point of the leader. The dynamical model for the agents, relating the control signal $u_k(t)$ of the k -th vehicle to its position $y_k(t)$ on the roadway, is given by

$$\dot{y}_k = v_k, \quad \dot{v}_k = f(v_k) + u_k; \quad (2a)$$

$$y_k(0) = -\sum_{j=0}^k \ell_j, \quad v_k(0) = 0. \quad (2b)$$

where $v_k(t)$ is the instantaneous speed of the k -th agent, $u_k(t)$ is its command signal and ℓ_k is the initial interspacing distance between the k -th agent and its predecessor in the string. Throughout the sequel we will use the notation

$$y_k = G_k \star u_k \quad (3)$$

to denote (especially for the graphical representations) the input-output operator G_k of the dynamical system from (2a), with the initial conditions (2b).

Assumption 2.4: The index “0” is reserved for the *leader vehicle*, the first vehicle in the string, for which we assume that there is no controller on board and consequently the command signal $u_0(t)$ will represent a *reference* signal for the entire formation.

We further define

$$z_k \stackrel{\text{def}}{=} y_{k-1} - y_k, \quad z_k^v \stackrel{\text{def}}{=} v_{k-1} - v_k \quad \text{for } 1 \leq k \leq n, \quad (4)$$

to be the interspacing and relative velocity error signals respectively (with respect to the predecessor in the string). By differentiating the first equation in (4) it follows that $\dot{z}_k(t) = z_k^v(t)$, therefore implying that constant interspacing errors (in steady state) are equivalent with zero relative velocity errors and also allowing to write the following time evolution for the relative velocity error of the k -th vehicle

$$\dot{z}_k^v = f(v_{k-1}) - f(v_k) + u_{k-1} - u_k. \quad (5)$$

III. A NOVEL DISTRIBUTED CONTROL ARCHITECTURE

The inherent difficulty in platooning control is rooted in the *nested* nature of the interdependencies between the regulated signals. Specifically, the regulated errors (e.g. interspacing errors or relative velocity errors) at the k -th agent depend on the regulated errors of its predecessor (the $(k-1)$ -th agent) and so on, such that by a recursive argument – going through all the predecessors of the k -th agent – they

and by employing the anti-symmetrical property of APFs [2, pp. 197] : $\nabla_{y_k} V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma) = -\nabla_{y_{k-1}} V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma)$ we get that

$$\frac{d}{dt} V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma) = 2\dot{z}_k^\top \nabla_{y_k} V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma). \quad (14)$$

Therefore from (11) it follows that

$$\begin{aligned} \frac{d}{dt} L_k(z_k(t), z_k^v(t)) &= \\ &= z_k^{v\top} \nabla_{y_k} V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma) + z_k^{v\top} \dot{z}_k^v \\ &= z_k^{v\top} (\nabla_{y_k} V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma) + \dot{z}_k^v) \\ &\stackrel{(10)}{=} z_k^{v\top} (f(v_{k-1}) - f(v_k)) - \beta_k z_k^{v\top} \dot{z}_k^v \\ &= z_k^{v\top} (f(v_{k-1}) - f(v_k)) - \beta_k z_k^{v\top} \dot{z}_k^v \end{aligned}$$

■

A preliminary framework in which the advantages of the closed-loop decoupling become apparent is illustrated by the following result. The *local* stabilization at the k -th agent is achieved irrespective of the dynamics involved at any other agents in the string and without invoking any stability arguments for the entire formation.

Proposition 3.2: Let the real function $f(\cdot)$ in (2a) be concave and differentiable, with its differential $f'(\cdot)$ upper-bounded by a given constant γ . Then, the controller (6) guarantees the stability of 0 as an equilibrium point of the decoupled system (10) as far as $\beta_k > \gamma$.

Proof: Let us notice

$$f(v_{k-1}) - f(v_k) \leq f'(v_{k-1})(v_{k-1} - v_k)$$

and since $f'(\cdot)$ is upper-bounded by a given constant γ , we get

$$f(v_{k-1}) - f(v_k) \leq \gamma z_k^v.$$

Therefore, along the trajectory of the closed-loop error system (10) the following holds

$$\dot{z}_k^v \leq (\gamma - \beta_k) z_k^v - \nabla_{y_k} V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma).$$

Using Gronwall's Lemma the stability of 0 as an equilibrium point of the decoupled system (10) is ensured by the stability of 0 as an equilibrium point for

$$\dot{z}_k^v = (\gamma - \beta_k) z_k^v - \nabla_{y_k} V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma). \quad (15)$$

The derivative of the Lyapunov function L_k defined in (11) along the trajectory of (15) can be computed following the prof of Lemma (3.1) as

$$\frac{d}{dt} L_k(z_k(t), z_k^v(t)) = (\gamma - \beta_k) z_k^{v\top}(t) z_k^v(t).$$

The proof ends by remarking that $\beta_k > \gamma$ guarantees that $\frac{d}{dt} L_k(z_k(t), z_k^v(t)) < 0$. ■

IV. MAIN RESULT

As we will show in this section, our methodology needs only information from the predecessor without employing any information from the leader agent. While the control gains are strictly related to the reactivity of the system (*i.e.* faster systems needs higher controller gains) our scheme does not require making the leader's information (instantaneous speed or acceleration) available to all other vehicles in the string (the virtual leaders from [2]), rendering our approach more suited to practical platooning applications. Our directed communications scheme, necessitates a minimal information exchange and sensing radius for all agents (each agent performs measurements and receives information only with respect to its predecessor). The following result is the main result of this Section, as it delineates a “decoupling” property of the closed-loop dynamics, achieved by the (6) type control policy along with velocity matching and collision avoidance.

Theorem 4.1: If the function $f(\cdot)$ from (2a) satisfies the global Lipschitz-like condition [2, Assumption 1]

$$(v_2 - v_1)^\top (f(v_2) - f(v_1)) \leq \alpha \|v_2 - v_1\|_2^2, \quad \forall v_1, v_2 \quad (16)$$

then for all type (6) control laws with that $\beta_k > \alpha$ the following hold:

(A) Given the Lyapunov function L_k introduced in (11), *local* to the k -th agent, the sub-level sets $\Omega_c^k \stackrel{def}{=} \{(z_k, z_k^v) | L_k \leq c, \text{ with } c > 0\}$ of L_k are compact and they represent forward invariant sets for the *local* closed-loop dynamics (10) of the k -th vehicle.

(B) The controller (6) guarantees velocity matching and collision avoidance. Furthermore, considering $c = 2L_k(z_k(0), z_k^v(0))$ there exists η_c such that

$$\|y_k - y_{k-1}\|_2 > \eta_c, \forall t \geq 0.$$

Therefore, a pre-specified safety distance can be imposed by the initial conditions.

Proof: (A) We show that the *local* sub-level sets $\Omega_c^k \stackrel{def}{=} \{(z_k, z_k^v) | L_k \leq c, \text{ with } c > 0\}$ of L_k are compact. Note that $L_k < c$ implies that $\|z_k^v\| < 2c$ and $V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma) < 2c$. Since $V_{k,k-1}$ is radially unbounded this implies that $\|z_k\|_\sigma$ is bounded and consequently $\|z_k\|_2$ is bounded. Therefore $\Omega_c^k \subset \mathbf{R}^{2\dim(y_k)}$ is a bounded set. Moreover due to continuity of $\|\cdot\|_\sigma$ and $L_k(\cdot)$ one obtains that Ω_c^k is a closed set. Precisely Ω_c^k is the pre-image of a closed set through a continuous function. In the Banach space $\mathbf{R}^{2\dim(y_k)}$ it therefore holds that Ω_c^k is closed and bounded thus Ω_c^k is compact. Furthermore, point (A) and the Lipschitz-like assumption (16) on $f(\cdot)$ implies that

$$\frac{d}{dt} L_k(z_k(t), z_k^v(t)) \leq (\alpha - \beta_k) z_k^{v\top}(t) z_k^v(t)$$

along the trajectories of (10). Therefore it suffices to choose the controller gain $\beta_k > \alpha$ in order to guarantee that $\frac{d}{dt} L_k < 0$ along the trajectories of (10) and that Ω_c^k is a forward invariant set for the decoupled closed-loop system (10), local to the k -th vehicle.

(B) From the properties of the APF it follows that

$V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma) \rightarrow \infty$ when $\|y_k - y_{k-1}\|_2 \rightarrow 0$. Consequently,

$$\forall c > 0, \exists \eta_c > 0 \text{ such that} \quad (17)$$

$$V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma) > c, \forall \|y_k - y_{k-1}\|_2 < \eta_c.$$

Denote with $c \stackrel{\text{def}}{=} 2L_k(z_k(0), z_k^v(0))$ and so for $\beta_k > \alpha$ it holds that Ω_c^k is forward invariant yielding $L_k(z_k(t), z_k^v(t)) \leq \frac{c}{2}, \forall t \geq 0$. This implies that $V_{k,k-1}(\|y_k - y_{k-1}\|_\sigma) < c, \forall t \geq 0$ and using (17) we conclude that $\|y_k - y_{k-1}\|_2 > \eta_c, \forall t \geq 0$. It is noteworthy that η_c is implicitly defined by c , which in turn depends on the initial condition. Therefore, the interspacing distance η_c between $(k-1)$ -th and k -th vehicles is imposed by the initial condition $(z_k(0), z_k^v(0))$. Finally, by employing LaSalle's invariance principle we conclude that the Lyapunov function L_k converges asymptotically to its minimum (i.e. $\frac{d}{dt}L_k = 0$) and consequently z_k^v converges to 0. Therefore, the velocity matching is guaranteed. ■

Proposition 4.2: Given $L_k(\cdot, \cdot)$ as introduced in (11), the string formation's steady-state configuration is attained at the minimum of the following formation-level Lyapunov function:

$$L(z(t), z^v(t)) \stackrel{\text{def}}{=} \frac{1}{2} \sum_{k=1}^n L_k(z_k(t), z_k^v(t)) \quad (18)$$

which coincides component-wise with the minima of the Lyapunov functions (11) *local* to the k -th agent. Furthermore, the level sets of L given by $\Omega_c \stackrel{\text{def}}{=} \{(z, z^v) | L \leq c, \text{ with } c > 0\}$ are compact and they represent forward invariant sets for the closed-loop dynamics of the entire formation, as given in (2) and (6) with $1 \leq k \leq n$. Consequently, velocity matching and vehicles' collision avoidance are achieved, *without the need for inserting exact leader information in the formation* (virtual leaders) while maintaining a safe interspacing distance.

Proof: It follows from the definition of (18) and Lemma 3.1 that along the trajectories of (2) and (6) one has

$$\frac{d}{dt}L = \sum_{k=1}^N z_k^{v\top} (f(v_k) - f(v_{k-1})) - \sum_{k=1}^N \beta_k z_k^{v\top} z_k^v. \quad (19)$$

Let us notice that Ω_c is a finite cartesian product of the compacts Ω_c^k , thus Ω_c is compact. Furthermore, designing the local controllers as in Theorem 4.1 it follows that Ω_c is a forward invariant set for the closed-loop dynamics of the entire formation. Consequently, without the need for inserting exact leader information in the formation, one guarantees the velocity matching and moreover from point **(B)** in Theorem 4.1 the vehicles maintain a strictly positive interspacing distance. ■

V. CONCLUSIONS

We have introduced a novel distributed control architecture for a class of nonlinear dynamical agents moving in the string formation, while guaranteeing trajectory tracking (with respect to the leader agent) and collision avoidance. The performance of the proposed scheme is entirely scalable with

respect to the number of vehicles in the string. The scalability is a consequence of the complete “decoupling” of a certain bounded approximation of the closed-loop equations, that allows the study of the closed-loop stability by performing solely individual, local analyses of the closed-loops stability for each agent, which in turn guarantees the aggregated stability of the overall formation.

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